Exclusivity-Consistency Regularized Multi-view Subspace Clustering

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Section 1. Problem:

Multi-view subspace clustering aims to partition a set of unlabeled multi-source data into their underlying groups.

- Many works prefer to learn a common representation, ignoring the complementary information between different views.
- Existing works tend to execute the subspace learning and spectral clustering in two separated steps, without consideration of the fact that these two steps highly depend on each other.

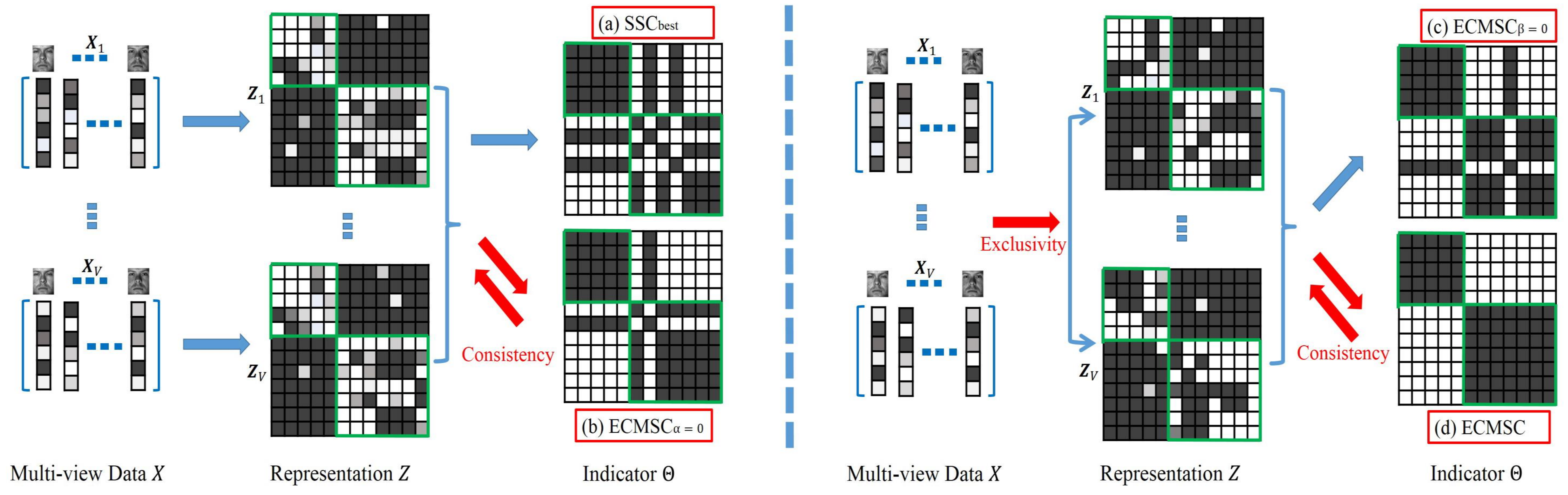
Contributions:

To overcome the above shortcomings, we propose a novel multi-view clustering algorithm namely ECMSC.

- A novel position-aware exclusivity term is proposed to effectively exploit the complementary information between different representations.
- An indicator consistent term is employed to advocate the label consistency among the complementary representations.

Section 2. Our Method (ECMSC):

- Compared to the ***value-aware*** Hilbert-Schmidt Independence Criterion (HSIC) [2], we introduce a novel ***position-aware*** exclusivity term, which can effectively avoid the scale issue of element values in different representations. Moreover, an indicator consistency term is proposed to unify the processing of subspace clustering.



Definition:

- **Exclusivity:** Exclusivity between two matrix $\mathbf{U} \in \mathbb{R}^{n\times n}$ and $\mathbf{V} \in \mathbb{R}^{n\times n}$ is defined as $\mathcal{H}(\mathbf{U},\mathbf{V}) = \|\mathbf{U}\odot\mathbf{V}\|_0 = \sum_{i,j}(u_{ij}\cdot v_{ij} \neq 0)$, where $\odot$ denotes the Hadamard product (i.e., element-wise product) .

Highlights:

- The exclusivity term encourages two matrix $\mathbf{U}$ and $\mathbf{V}$ to be diverse.
- The exclusivity term is position-aware. If the position (i,j) of $\mathbf{U}$ is not zero, the same position (i,j) of $\mathbf{V}$ is enforced to be zero.

Exclusivity-Consistency:

- Representation Exclusivity: To make the exclusivity of different representations computationally tractable, we relaxed it as:

$$\min_{\mathbf{Z}_v} \mathcal{H}(\mathbf{Z}_v,\mathbf{Z}_w) = \min_{\mathbf{Z}_v} \|\mathbf{Z}_v\odot\mathbf{Z}_w\|_1$$

- Indicator Consistency: Knowing that the goal of clustering is to classify a point into only one cluster, we introduce the label consistency term as:

$$\min_{\mathbf{F}} \|\mathbf{Z}_v\odot\mathbf{\Theta}\|_1$$

where $\mathbf{\Theta}$ is the common indicator matrix for all the views.

The Objective Function:

To consider the exclusivity of different representations and the consistency of indicators into one framework:

$$\min_{\mathbf{F},\mathbf{Z}_1,\dots,\mathbf{Z}_V} \sum_{v=1}^{V} \|\mathbf{E}_v\|_1 + \lambda_1\|\mathbf{Z}_v\|_1 + \lambda_2 \underbrace{\sum_{w\neq v}\|\mathbf{Z}_v\odot\mathbf{Z}_w\|_1}_{\textbf{Exclusivity}} + \lambda_3 \underbrace{\|\mathbf{Z}_v\odot\mathbf{\Theta}\|_1}_{\textbf{Consistency}}$$

$$\text{s.t.}\quad \forall\, v,\quad \mathbf{X}_v = \mathbf{X}_v\mathbf{Z}_v + \mathbf{E}_v,\quad \mathrm{diag}(\mathbf{Z}_v) = 0,\quad \mathbf{F}^T\mathbf{F} = \mathbf{I}$$

where $\theta_{ij} = \frac{1}{2}\left\|\mathbf{f}^i - \mathbf{f}^j\right\|_2^2$.

Algorithm:

We propose a solution by solving the two sub-problems alternatively:

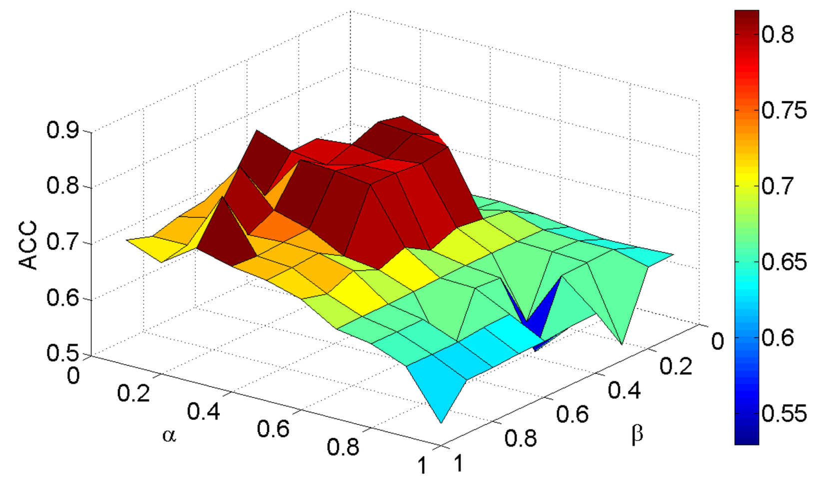
- Given $\mathbf{F}$, compute each exclusive representation $\mathbf{Z}_v$ and the corresponding residual $\mathbf{E}_v$ by ADMM algorithm.
- Given $\mathbf{Z}_v$ and $\mathbf{E}_v$, find the consistent indicator $\mathbf{F}$ by spectral clustering.

Section 3. Experiments:

- Given a set of unlabeled data with multi-view features, the ECMSC algorithm will directly output the clustering results.
- **Extended Yale-B Results**:

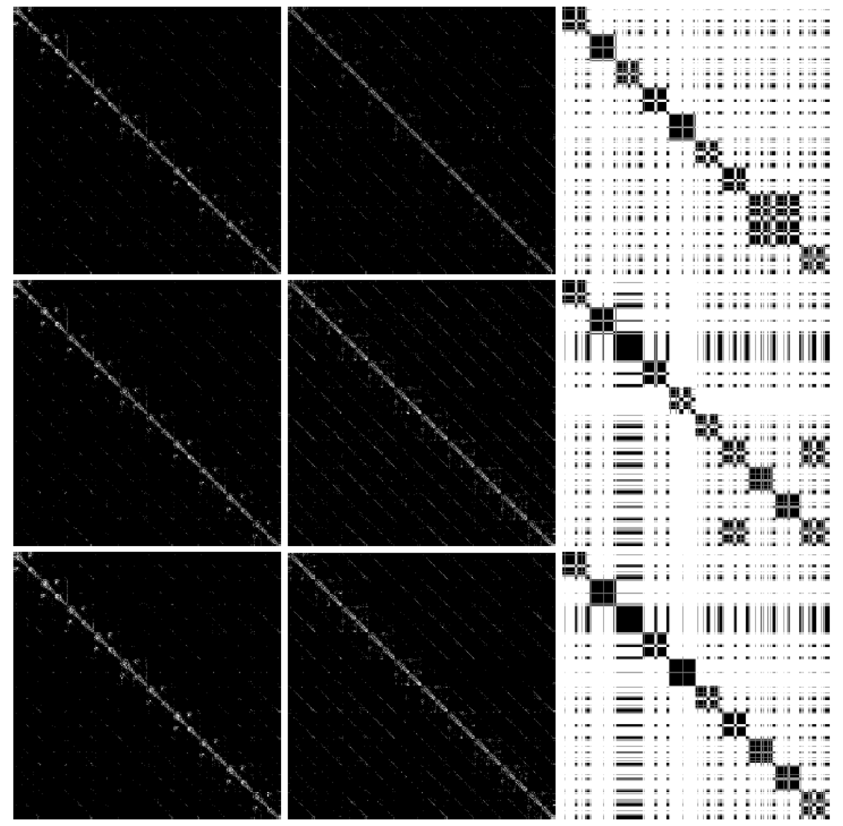
	Method	NMI	ACC	ARI	F-score	Precision	Recall
Single	SPC_{best}	0.360±0.016	0.366±0.059	0.225±0.018	0.303±0.011	0.296±0.010	0.310±0.012
	SSC_{best}	0.534±0.003	0.587±0.003	0.430±0.005	0.487±0.004	0.451±0.002	0.509±0.007
	$S3C_{best}$	0.542±0.010	0.391±0.012	0.415±0.007	0.492±0.004	0.417±0.005	0.487±0.009
Multiple	$FeaCon_{PCA}$	0.152±0.003	0.232±0.005	0.069±0.002	0.161±0.002	0.158±0.001	0.64±0.002
	Min-Dis	0.186±0.003	0.242±0.018	0.088±0.001	0.181±0.001	0.174±0.001	0.189±0.002
	Co-Reg SPC	0.151±0.001	0.224±0.000	0.066±0.001	0.160±0.000	0.157±0.001	0.162±0.000
	ConReg SPC	0.163±0.022	0.216±0.019	0.072±0.012	0.164±0.010	0.163±0.010	0.165±0.011
	LT-MSC	0.637±0.003	0.626±0.010	0.459±0.030	0.521±0.006	0.485±0.001	0.539±0.002
	DiMSC	0.635±0.002	0.615±0.003	0.453±0.000	0.504±0.006	0.481±0.002	0.534±0.001
Proposed	$ECMSC_{\alpha=0}$	0.719±0.011	0.692±0.013	0.492±0.008	0.548±0.007	0.481±0.004	0.691±0.006
	$ECMSC_{\beta=0}$	0.708±0.009	0.678±0.010	0.482±0.011	0.530±0.009	0.487±0.004	0.672±0.011
	ECMSC	**0.759±0.012**	**0.783±0.011**	**0.544±0.008**	**0.597±0.010**	**0.513±0.009**	**0.718±0.006**

- **Parameters Effects**:



- Inspired by previous works [25,18], we set $\lambda_1 = \eta^{1-t}$, $\lambda_2 = \alpha$ and $\lambda_3 = \beta\eta^{t-1}$, where $\eta = 1.2$ and $t = \{1,2,\dots,T\}$ is the iteration index. α is to control the representation exclusivity term. β is to balance the indicator consistency term.

- **Representation Visualization:**



- **From left to right**: The columns are visualization of subspace representations $\mathbf{Z}_1$, $\mathbf{Z}_2$ and the indicator matrix $\mathbf{\Theta}$.
- **From top to bottom**: The rows are the results of $ECMSC_{\alpha=0}$(ACC=0.701), $ECMSC_{\beta=0}$(ACC=0.689) and ECMSC(ACC=0.781), respectively.

Code: http://www.cbsr.ia.ac.cn/users/xiaobowang/